Use of the uniform, time dependent temperature distribution in another analysis[4] has lead to relatively simple expressions for the bond stress with a viscoelastic propellant.

References

[1] Ungar, E. E. and Shaffer, B. W., "Thermally induced bond stresses in case-bonded propellant grains," ARS J. **30**, 366–368 (1960).

[2] Krause, I. and Shaffer, B. W., "Thermal stresses in spherical case-bonded propellant grains," Trans. Am. Soc. Mech. Engrs. B, 144–148 (February 1962).

[3] Zwick, S. A., "Thermal stresses in an infinite hollow case-bonded cylinder," Jet Propulsion **27**, 872–876 (1957).

[4] Shaffer, B. W. and Levitsky, M., "Thermal bond stresses in case-bonded viscoelastic propellant disks," J. Aerospace Sci. **29**, 827–833 (1962).

[5] Schneider, P. J., *Conduction Heat Transfer* (Addison-Wesley Publishing Co., Reading, Mass., 1955), p. 230.

[6] Heisler, M. P., "Temperature charts for induction and constant temperature heating," Trans. Am. Soc. Mech. Engrs. **69**, 741–752 (1947).

Comment on "Insulation Requirements for Long-Time Low-Heat Rate Environments"

D. M. TELLEP* AND T. D. SHEPPARD†
Lockheed Missiles and Space Company, Sunnyvale, Calif.

IN a previous note,[1] the authors discussed the problem of insulation requirements for long-time low-heat rate environments. With several simplifying assumptions, the problem was reduced to that of heat conduction in a slab. It was stated that the solution to the diffusion equation for the conditions of interest could not be found in standard heat conduction tests, and a solution was presented. It has subsequently been brought to the authors' attention that the solution presented in Ref. 1 is available in standard texts, such as Ref. 2. A solution to the same problem using Laplace transforms is also presented in Ref. 3. The latter solution is of a more convenient form than that obtained using Fourier series and is shown in Fig. 1. It is noted that the required insulation weight per unit area appears only in the ordinate. Thus, the insulation weight per unit area may be obtained directly without the iteration process demanded by the figure in the original note.

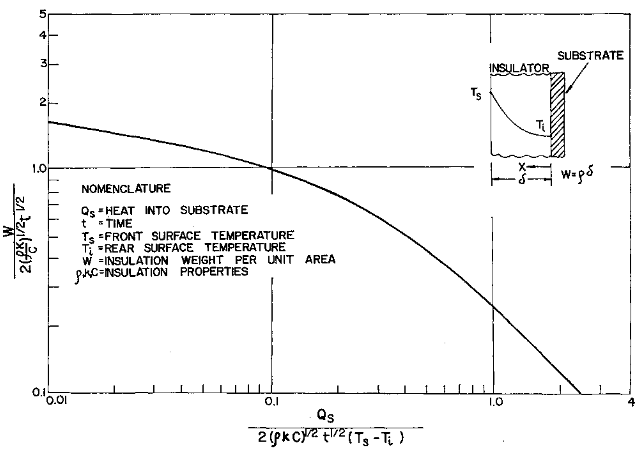

Fig. 1 Heat transmitted to rear surface of slab whose surfaces are maintained at constant temperatures.

Received August 29, 1963.

* Manager, Launch and Entry Thermodynamics. Associate Member AIAA.

† Research Specialist.

References

[1] Tellep, D. M. and Sheppard, T. D., "Insulation requirements for long-time low-heat rate environments," AIAA J. **1**, 1670–1671 (1963).

[2] Churchill, R. V., *Fourier Series and Boundary Value Problems* (McGraw-Hill Book Co. Inc., New York, 1948), 1st ed., pp. 109–110.

[3] Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat In Solids* (Oxford University Press, England, 1947), 1st ed., p. 252.

Correction and Addition to "Survey of Current Literature on Satellite Lifetimes"

B. H. BILLIK*
Aerospace Corporation, El Segundo, Calif.

IT has recently been brought to the attention of this author that Table 2 of Ref. 1 contains erroneous altitude data on the satellites noted and incorrect lifetime estimates.† Corrected perigee and apogee altitudes are (from top to bottom)

$$h_p = 95, 86, 123, 118, 120, 90, 104 \text{ naut miles}$$

$$h_a = 797, 526, 190, 391, 466, 477, 916 \text{ naut miles}$$

The estimated lifetimes associated with the above altitudes, however, have not been computed at this time.

Equations (66) and (67), which were hailed as rather general expressions for obtaining the number of revolutions and the time to decay, contain both the final radius and eccentricity of the orbit. In order to evaluate these expressions, an equation relating these quantities is needed. Reference 2 shows that the appropriate relationship is

$$\frac{e_1}{1+e_1}\,\rho_{p_1}\,j_1\left(\frac{\beta r_{p_1} e_1}{1+e_1}\right) = \frac{e_0}{1+e_0}\,\rho_{p_0}\,j_1\left(\frac{\beta r_{p_0} e_0}{1+e_0}\right) \qquad (1)$$

where the terms are defined in Ref. 1.

References

[1] Billik, B., "Survey of current literature on satellite lifetimes," ARS J. **32**, 1641–1650 (1962).

[2] Nonweiler, T., "Perturbations of elliptic orbits by atmospheric contact," J. Brit. Interplanet. Soc. **16**, 368–379 (1958).

Received September 18, 1963.

* Member of the Technical Staff. Member AIAA.

† The lifetimes shown in the table do not correspond to the satellite data listed.

Optimization of System Reliability

DAVID H. SILVERN*
David H. Silvern and Company, Engineering Consultants, North Hollywood, Calif.

WHILE conducting an analysis of a space power system, this author came upon a criterion for optimizing system reliability which seemingly presented a new insight into

Received October 3, 1963.

* President.

this very elusive subject. The result was so simple and useful that it seemed unlikely to the author that the concept was not published and in general use. Surprisingly, it was learned, after a discussion of this author's approach with a number of reliability experts, including analysts in the reliability departments of Grumman, RCA, Marquardt, United Airlines, Planning Research Corporation, and others, that this might not be so. The idea was new to them. Not being able to find a reference in the scattered reliability literature, after an admittedly cursory search, this idea is presented in the hope that it will prove interesting and useful, if not really new.

Let us start, as did the author originally, with a very straightforward problem. Let us assume a system made up of m subsystems, each of which has n_i redundant elements. Now the failure of any subsystem will abort the mission, but this will require the failure of all n_i of its redundant elements. If the weight of each of these elements is w_i, and each element has a reliability of r_i, what is the optimum number of elements for each subsystem required to minimize the weight while maintaining a given system reliability?

The reliability R and weight W of the system can be written as follows:

$$R = [1 - (1 - r_1)^{n_1}][1 - (1 - r_2)^{n_2}] \ldots [1 - (1 - r_m)^{rm}]$$

$$W = n_1w_1 + n_2w_2 + \ldots + n_mw_m$$

Let us allow the n's to vary and assume, for the moment, that their values are large enough so that they can be varied continuously (rather than step-wise) without a large error. By using the method of Lagrange, one obtains the following set of equations for the optimum weight distribution:

$$\frac{\partial(R - W\lambda)}{\partial n_i} = \frac{(1 - r_i)^{n_i} \log(1 - r_i)][1 - (1 - r_1)^{n_1}] \ldots [1 - (1 - r_m)^{n_m}]}{[1 - (1 - r_i)^{n_i}]} + \lambda w_i = 0$$

Upon dividing each equation by Rw_i we obtain the following:

$$\frac{-\lambda}{R} = \frac{(1 - r_1)^{n_1} \log(1 - r_1)}{w_1[1 - (1 - r_1)^{n_1}]} = \frac{(1 - r_2)^{n_2} \log(1 - r_2)}{w_2[1 - (1 - r_2)^{n_2}]} = \frac{(1 - r_i)^{n_i} \log(1 - r_i)}{w_i[1 - (1 - r_i)^{n_i}]}$$

It is evident that, for any system which contains a multiplicity of elements, the reliability of any given sub-system must be reasonably close to one, and the brackets in the denominator can be eliminated with little loss in accuracy. Let us multiply the numerator and denominator of the ith equation by n_i:

$$\frac{(1 - r_1)^{n_1} \log(1 - r_1)^{n_1}}{n_1 w_1} = \frac{(1 - r_2)^{n_2} \log(1 - r_2)^{n_2}}{n_2 w_2} = \frac{(1 - r_i)^{n_i} \log(1 - r_i)^{n_i}}{n_i w_i}$$

The weight of the elements making up the ith subsystem will then be $W_i = n_i w_i$ and the reliability $R_i = 1 - (1 - r_i)^{n_i}$.

Let us define a new term, the "unreliability," $U_i = 1 - R_i$. The equations can be simply written

$$\frac{U_1 \log U_1}{W_1} = \frac{U_2 \log U_2}{W_2} = \frac{U_i \log U_i}{W_i}$$

One last approximation remains. For highly reliable, multiple element systems, the U_i terms will be very small numbers, perhaps varying between 10^{-2} and 10^{-6}. The logarithms of the factors will be in the ratio of 3, at the most. The number represented by the unreliability $(1 - R_i)$ is not known within an order of magnitude so that the elimination of a factor of one-half this order of magnitude is of small significance. We can then write the following equation, which defines the optimum distribution of weight between subsystems:

$$\frac{U_1}{W_1} = \frac{U_2}{W_2} = \ldots = \frac{U_m}{W_m}$$

or, to state the essence as a rule, "unreliability" per unit weight must be close to a constant in every part of a system to attain optimum reliability per pound for the over-all system.

This solution pertains to systems whose reliability was increased by adding redundancy, which makes reliability a very strong function of weight. Most reliability factors (r_i) *are also* strong functions of weight, and the rule will be applicable for systems without redundancy.

For example, if the reliability of the ith subsystem varies as its weight to the k_ith power, the resulting optimization equations are the same with $\log U_i$ replaced by k_i. As with the logarithms of the previous case, the k's may be canceled out provided they do not vary by more than a factor of three—and the same final result obtains.

In ground systems, where cost rather than weight is the criterion of goodness, it can be shown (by assuming the cost per part is linear with n_i) that constant "unreliability" per dollar will optimize the system.

The significance of the rule can be understood as follows: In order to increase system reliability, it is sometimes advantageous to increase the weight of a light part or add a redundant component even if it already has a very high reliability, rather than do the same with a heavy part of low reliability. The gain in over-all system reliability *per pound of additional weight* can be greater. The rule tells when this is true, and when it is not, to a degree of accuracy which is probably better than the reliability data of the individual elements.

A certain number of assumptions and approximations were made in the determination of this rule. But given the "unreliability" of reliability information and the expense of setting up an optimization procedure during the preliminary design phase when the system is changing from day to day, this simple formulation becomes more than a rule of thumb. It is a rationale for many decisions which are now made by guess and by deadline.